Lab 3: Magnetic Ball Levitator

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***Abstract*—In this lab, we will levitate a metal ball by using an electromagnet. The system is highly nonlinear and unstable, and will be linearized using an affine linearization on a small range around a designated trim point. Once a linearized model of the system is obtained, we will create full-state and PID controllers in order to make the system levitate the ball at a stable position.**

***Index Terms*—xPC target, electromagnet, magnet, full state, PID, PD, affine linearization, nonlinear systems**

1. Introduction/Objectives

IN THIS LAST LAB ACTIVITY, we will analyze an unstable and nonlinear system. Using affine linearization, we will attempt to make a ball metal levitate by using an electromagnet and an optical sensor. To achieve this goal, we will utilize a few different controller architectures, such full-state and PID controllers.

A. Experiment Objectives

The first goal of this lab activity is to obtain system parameters and calibrate all of the experimental apparatus. Once this is done, the next step will be to derive a relationship between magnetic force and distance to the magnet, and then create a simulated model of the plant. After this step is achieved, an affine linearization is to be performed on the system to allow the use of linear controllers to tackle the issue. Finally, full-state, PD and PID controllers are to be implemented in an attempt to make the ball levitate.

2. Experimental Apparatus

IN THIS EXPERIMENT, we worked with a MATLAB/Simulink host computer environment, as well as an xPC target environment. The experimental apparatus consisted of an optical sensor, a force transducer, a position stand, and an electromagnet. The electromagnet and optical sensor were mounted in a wooden stand to ensure a constant distance between both of the devices.

3. System Modeling

THE OBJECTIVE of this lab activity is to model and analyze an electromagnet and an optical sensor. The results obtained from these analyses will then be used in conjuction with different controllers to make the metal ball levitate.

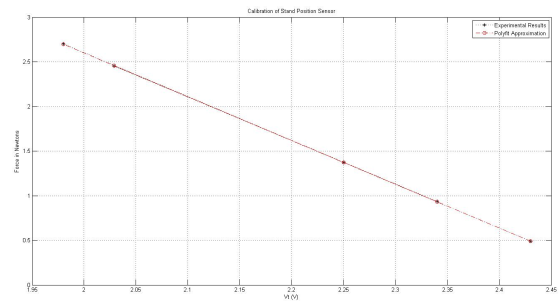

Fig. 1. Experimental results of the force transducer and polyfit approximation

A. Mathematical Model

Many of the physical relationships in this experiment were highly non-linear, and the best way to proceed was to empirically study their behavior. As an initial experiment, we used a mounted force stand transducer to determine the force exerted by the magnet at different z-positions (vertical positions) of the ball.

Fig. 1 shows the experimental resuts obtained from the calibration of the force stand transducer, and the linear polyfit approximation made with those results. Using the polyfit function in MATLAB, we obtained the following results:

$$F_{\text{poly}} = 12.4076\ - 4.9037\text{Vt}$$

Where Vt is the voltage outputted by the transducer, and F the inputted force.

In a similar manner, the voltage output from the position sensor was collected at different z-locations of the ball. The results are shown in Fig. 2.

As can be seen from these results, there is a linear correspondence between position and voltage. On the other hand, the optical sensor exhibits cubic behavior. We can also see that there is a small range where the optical sensor can be approximated as a linear function from 0.25 to 5V and from 23 to 28 mm from the magnet. To avoid further complications, the point where the ball is to be levitated will be placed within this distance interval. See Fig. 3 for the optical sensor data.

B. Parameters Determination

To determine the forces that the magnet should exert in order to levitate the ball, a relationship between force and the distance from the magnet should be established. The magnet was given specific input voltages, and at each voltage we performed a sweep to record the magnetic force at various distances to the magnet. The results are shown in Fig. 4.

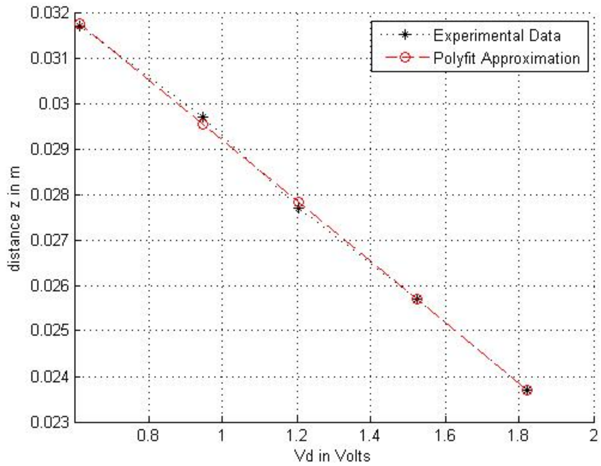

Fig. 2. Experimental results of the position sensor and polyfit approximation

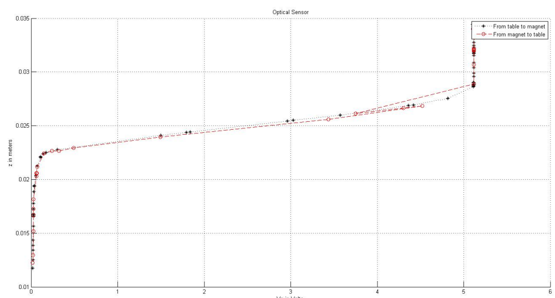

Fig. 3. Experimental results of the optical sensor

After performing the magnetic sweeps, we reduced the data obtained from the force transducer and the position sensor in order to establish the relationship between force and distance. The reduced data can be seen in Fig. 5 .

As we can see, the plots in Fig. 5 show an exponential evolution. For this reason, we can assume that for each plot:

$$F_m\left(z, V_m\right) = \alpha \mathrm{z}^{\beta}$$

$$\log\left(F_m\right) = \log(\alpha) + \beta \log(z)$$

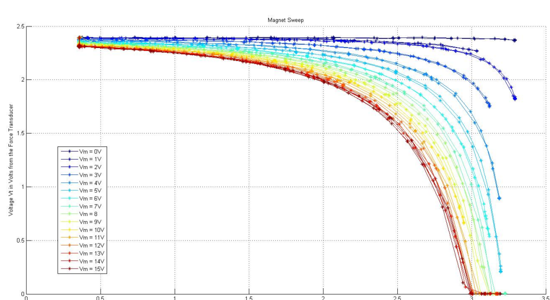

Fig. 4. Experimental results of the magnets sweep

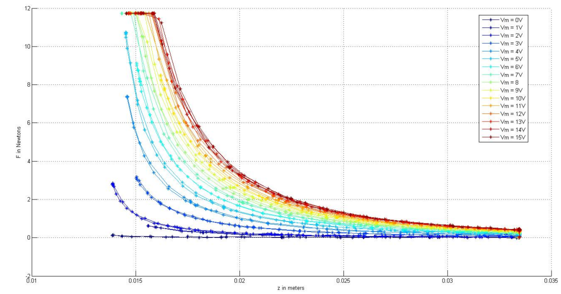

Fig. 5. Magnet sweep data reduced to show Force vs Distance

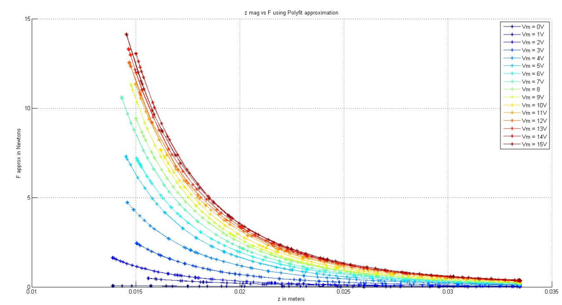

Fig. 6. Numeric approximation of magnetic force vs distance

Therefore, using Eq. 3-B, we can approximate the values of α and β, and then find a numeric approximation of the relationship between force and distance. This relationship is shown in Fig. 5.

C. Nonlinear State Space Representation

Since the magnetic force depends on two variables, the result is a family of non-linear curves. Rather than trying to derive a transfer function, we will tackle this problem by creating a 2D look-up table. Once this step is completed, a state-space representation of the model will be derived. Assuming the only forces acting on the ball are gravity and magnetic force, we can use Newton's 2nd law on the ball:

$$\Sigma \mathrm{F}_z = m\ddot{z}(t)$$

$$m\ddot{z}(t) = F_m - \mathrm{mg}$$

It is assumed that when the ball is levitating an equilibrium is reached between the two aforementioned forces. Therefore, if a trim point is to be selected it would be of the form:

$$\bar{z} = \begin{bmatrix} z(t) \\ \dot{z}(t) \end{bmatrix}$$

$$\bar{z}_{\text{trim}} = \begin{bmatrix} z_{\text{trim}} \\ 0 \end{bmatrix}$$

To find a trim point, we need to look at the situation where magnetic force and gravity cancel each other, and select a trim point then find its respective input voltage for the

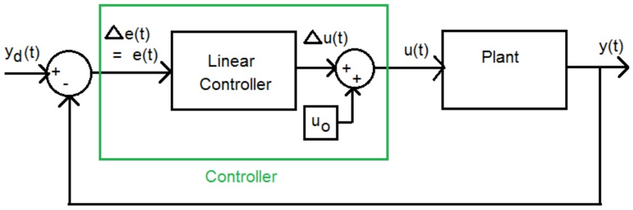

Fig. 7. Simulink model to be used with an affine linear approximation

electromagnet. We chose a distance of 26 mm for the trim point in order to be at the center of the linear range within the optical sensor's output.

$$\bar{z}_{\text{trim}} = \left[\begin{pmatrix} 0.026 \\ 0 \end{pmatrix}\right]$$

$$u_0 = -3.572V$$

The system is a highly non-linear one, so we performed an affine linearization. The affine linearization is based on the following assumptions:

$$f(x) \approx f(x_0) + \frac{(x - x_0)\,\delta\text{f}(x_0)}{\delta\text{x}}$$

$$\Delta\text{f}(x) = f(x) - f(x_o)$$

$$\Delta\text{f}(x) \approx \frac{(x - x_0)\,\delta\text{f}(x_0)}{\delta\text{x}}$$

Then the state-space affine linearization is reduced to:

$$\Delta\dot{\bar{z}} = \text{A}\Delta\bar{z} + \text{B}\Delta\bar{u}$$

$$\Delta\bar{y} = \text{C}\Delta\bar{x} + \text{D}\Delta\bar{u}$$

For this experiment the matrices A, B, C and D were obtained using the MATLAB function linmod().

D. Simulink Modeling

The Simulink model shown in Fig. 7 can be used after we first recognize the following equalities:

$$\Delta\text{z}(t) = z(t) - z_0$$

$$\Delta\text{u}(t) = u(t) - u_0$$

$$\Delta\text{y}_d(t) = y_d(t) - y_{d,0}$$

$$\Delta\text{y}(t) = y(t) - y_0$$

$$\Delta\text{e}(t) = e(t)$$

Following the model in Fig. 7, the idea is to create a plant simulation of the system by using the 2D look-up table. A linear controller can be used, so long as it doesn't deviate too much from the trim position, and the trim conditions are added or substracted as dictated by the derivation.

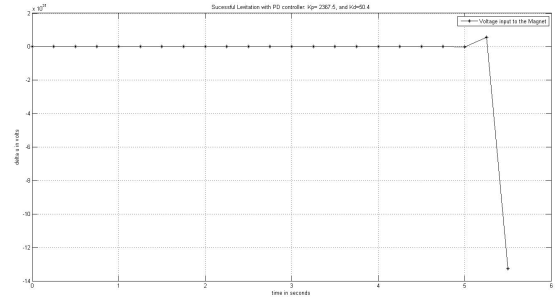

Fig. 8. Voltage output by the controller with respect to trim voltage Δu

4. Control System Design

For this type of system, we will study the performance of two highly effective linear control systems: the full-state feedback controller, and a PID controller.

A. Full State Feedback Controller

A full-state feedback controller works as a regulator, trying to keep the system fixed around a specific value. This is exactly what we hope to accomplish by levitating the ball. Since the full-state feedback controller basically uses a different gain for each state parameter of the system, it can be simplified as follows:

$$\dot{\bar{x}}(t) = A\bar{x}(t) + B\left(-K\bar{x}(t)\right)$$

$$\dot{\bar{x}}(t) = A - \text{BK}\bar{x}(t)$$

$$\dot{\bar{x}}(t) = A_{\text{cl}}\bar{x}(t)$$

It is important to know that the system is not fully controllable; this means that despite the use of a full-state feedback controller, not every parameter can be set as desired. However, the current objective of making the ball levitate is the same as fixing the ball in a pre-set position. Therefore, the K gain values all other parameters are unimportant, so long as the trim position and the corresponding gain are correct. For the first attempt to find gains, we used a pole placement method. By setting the eigenvalues, or answers to Eq. 4-A, to the arbitrarily chosen poles of -120, -110, and -100, the controller is expected to deliver the desired outputs, and to exhibit stability due to its strictly negative eigenvalues. The data obtained is shown in Fig. 8 and Fig. 9.

The graphs shown in Fig. 8 and Fig. 9 show voltages that the experimental setup will not be able to produce.

To acquire more optimal results, we used the LQR algorithm to select gains via the MATLAB function lqr(). The values of the matrices Q and R needed to compute the gains where chosen as identity matrices in both cases. In our tests, the ball stuck immediately to the magnet, making the test unsuccessful. The data we obtained is shown in Fig. 10 and Fig. 11.

Fig. 10 shows that on average, the controller outputs voltage in the range of 30V. This range is outside the limits of

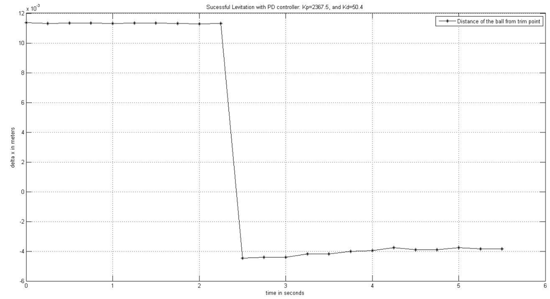

Fig. 9. Change in position of the ball with respect to trim position Δx

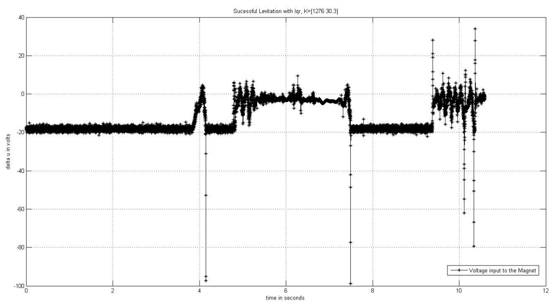

Fig. 10. Voltage output by the controller with respect to trim voltage Δu

the Kepco power amplifier. We calculated new gains, this time reducing the value of R to 0.0000001 to accomodate the amplifier limits. The new gains had values of K=[3864 3162.7]. The data obtained is shown in Fig. 12 and Fig. 13.

In this new case, the magnetic ball levitates briefly and then falls. It can be assumed that while the controller is operating correctly, it has insufficient control authority, i.e., enough magnetic force, to make the ball levitate. Changing the value of R once again, this time to 0.001, we obtain the gains of K=[1276.8 43.8]. This time the ball successfully levitates. The data obtained is shown in Fig. 14 and Fig. 15.

B. PD and PID Controllers

In the next step, the full-state feedback controller was replaced by a PD controller. In this specific scenario, the architecture of both controllers is exactly the same: therefore

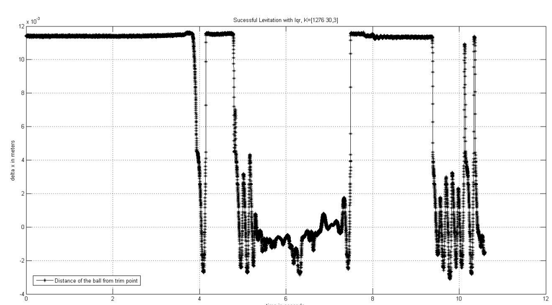

Fig. 11. Change in position of the ball with respect to trim position Δx

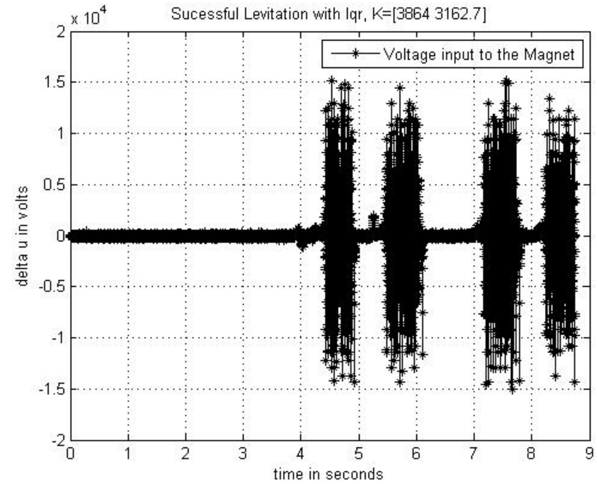

Fig. 12. Voltage output by the controller with respect to trim voltage Δu

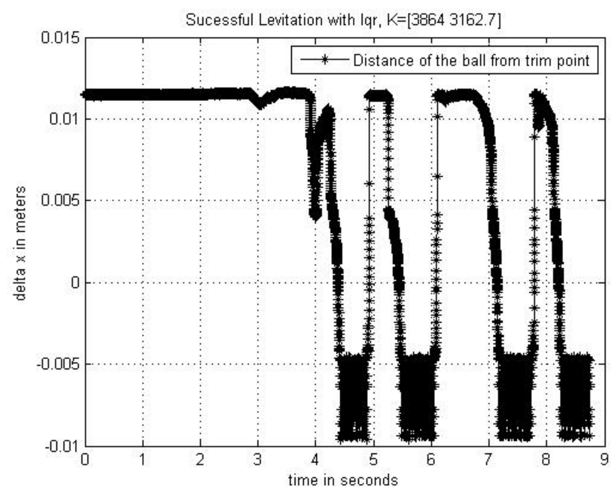

Fig. 13. Change in position of the ball with respect to trim position Δx

using the same gain values we should expect a similar result. By setting Kp = 1276.8 and Kd = 43.8, the ball levitates. The data obtained is shown in Fig. 16 and Fig. 17. You may see a photograph of our ball levitating in Fig. 18.

We empirically tweaked data to find a suitable integral gain, and settled on a gain of Ki = 10. When we add the integrator gain to turn the PD into a PID controller, it can be observed that the evolution of both parameters (Δx and Δu) is fairly similar either controller. The data obtained is shown in Fig. 19 and Fig. 20.

5. Conclusions

IN THIS LAB ACTIVITY, we studied and utilized methods to analyze and control a highly unstable, non-linear system. Each part of this laboratory has been extremely challenging, from obtaining the experimental parameters to developing a successful controler. This lab was based on the very important assumption that even non-linear systems can be approximated as linear on very small intervals. We simulated and studied a linear plant based on this hypothesis. Other important points

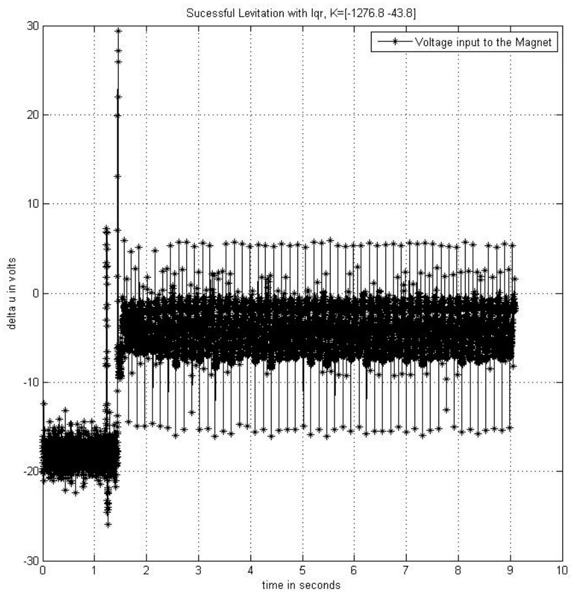

Fig. 14. Voltage output by the controller with respect to trim voltage Δu

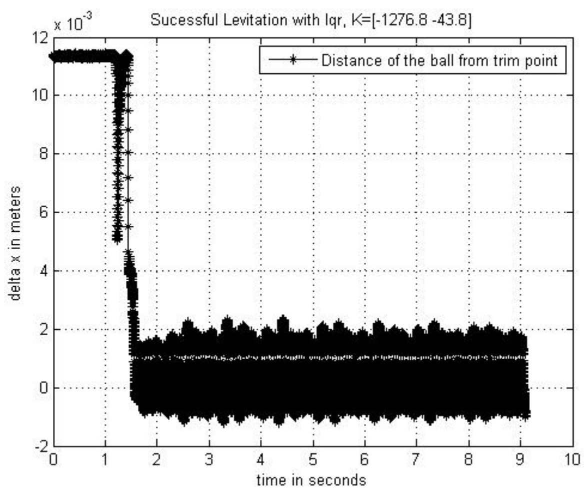

Fig. 15. Change in position of the ball with respect to trim position Δx

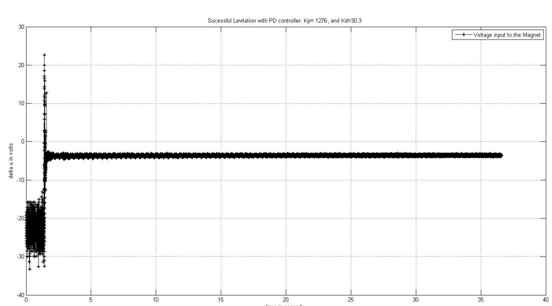

Fig. 16. Voltage output by the controller with respect to trim voltage Δu

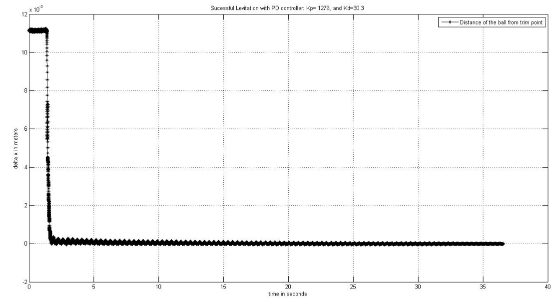

Fig. 17. Change in position of the ball with respect to trim position Δx

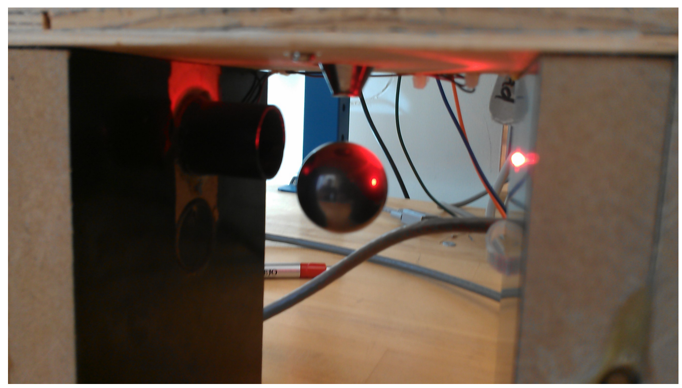

Fig. 18. Metal ball levitated using our PD controller.

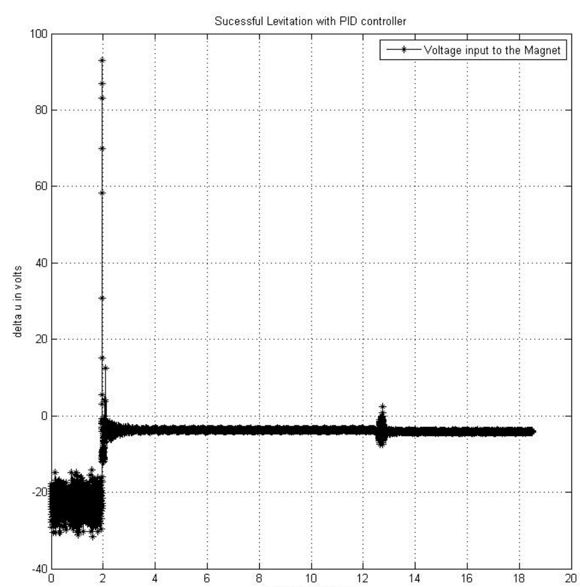

Fig. 19. Voltage output by the controller with respect to trim voltage Δu

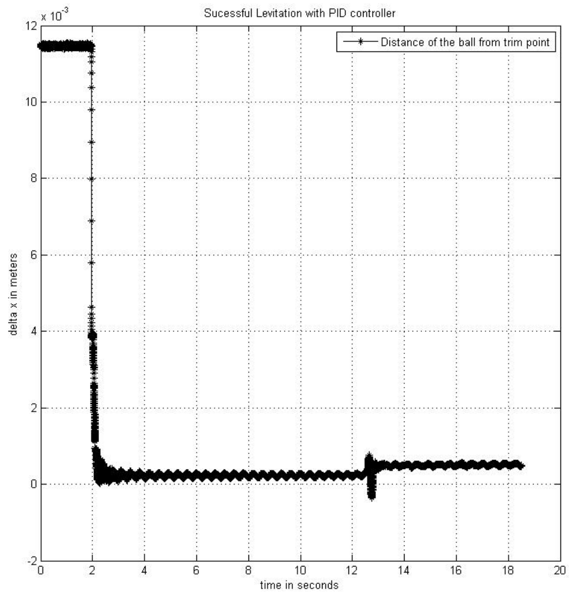

Fig. 20. Change in position of the ball with respect to trim position Δx

of this activity were the study and test of full-state feedback and PID controllers: while theoretically a full-state feedback controller can achieve any goal, the physical limits of the system can be a serious obstacle in its implementation. On the other hand, a PID controller, while using simpler logic, seems very effective in practical use. The main disadvantage of the PID controller is the lack of an effective method to calculate the desired gains. In the previous scenario, an optimization approach to the problem can be taken by using the LQR algorithm.

A. Further Work

We obtained rather successful results from our linearization and subsequent gain choice; however, it would be interesting to study this system as a non-linear one, and see if the application of non-linear control techniques could produce great stability and robustness. We did minor tests with regards to robustness–we were able to hang several small metal balls from our levitating ball without it falling–but a greater study of the robustness of the system, including the maximum disturbance force that could be applied while still keeping the system stable, would be of great interest for further work.